ENHANCING STATISTICS TEACHER EDUCATION WITH E-MODULES

# Supporting Inferential Reasoning through Modeling, Simulation, and Argumentation

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ESTEEM

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# Presentation Outline

- Considering Tasks that Support Inferential Reasoning
- Engage in Tasks
  - Collecting Cane Toads
  - Investigating $CO_2$ Emissions of Vehicles
  - Investigating Schoolopoly

# Inferences from Samples

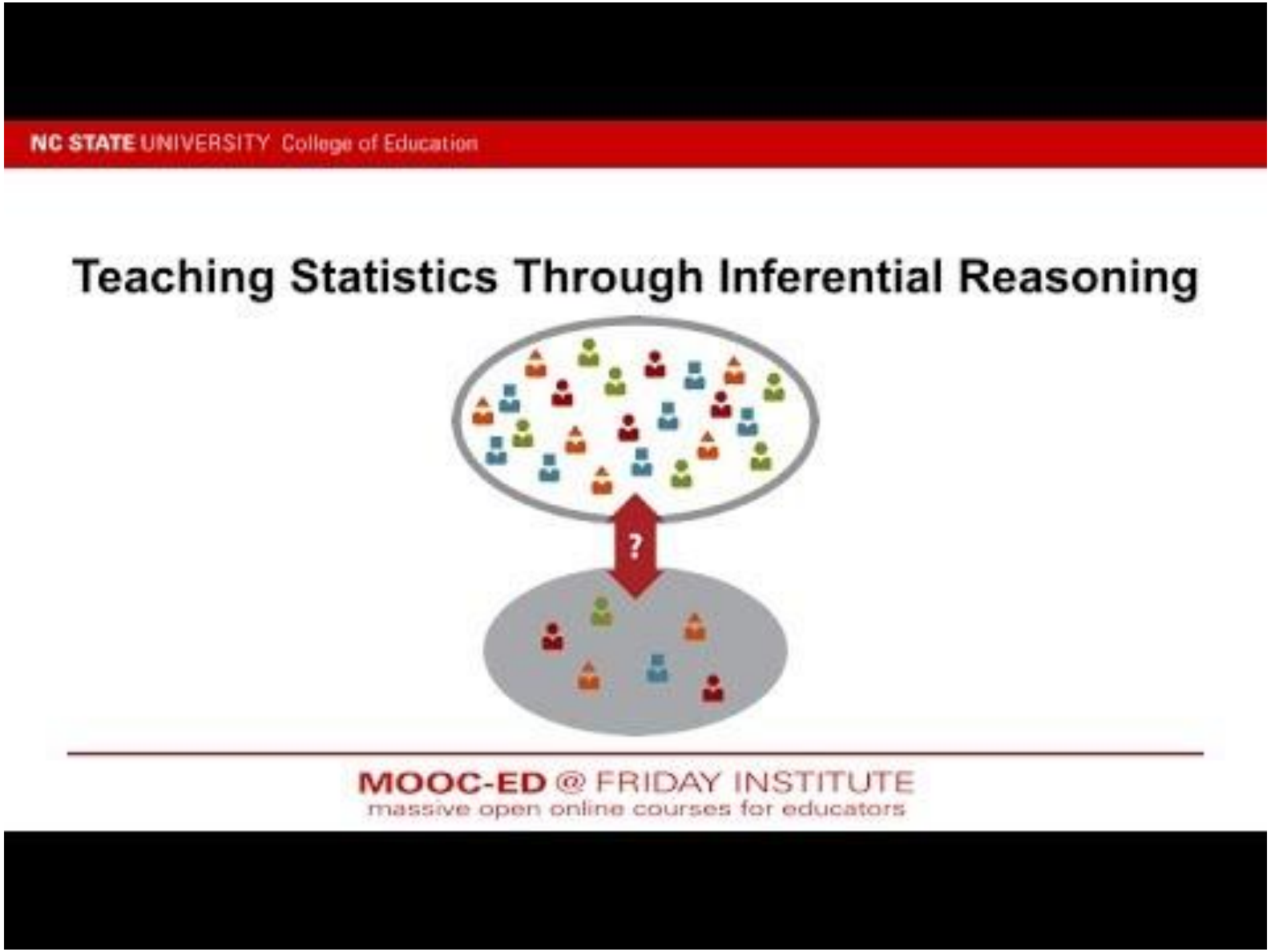
Inferences can be made from a **sample to a population** and from a **sample to a process that produced the sample** (Makar & Rubin, 2018)

- methods for drawing conclusions from data about the **population** or process from which the data are drawn (Cobb & Moore, 1997, p. 813)
- process of assessing strength of evidence concerning whether or not a set of observations is consistent with a **particular hypothesized mechanism that could have produced** those observations (Harradine, Batanero, & Rossman, 2011, p. 235)

# Consider

What are the characteristics of a task that provide opportunities to engage with inferential reasoning?

# Expert Panel Discussing Critical Components of IR Tasks



# Key Aspects of Inferential Reasoning

- Using rich and meaningful contexts
- Analyzing data with multiple representations and measures
- Considering variability
- Modeling and models
- Inferring beyond data with uncertainty

(Makar, 2013; Makar & Rubin, 2018, 2009; Lee, 2017; Rossman, 2008; Zieffler, Garfield, delMas, & Reading, 2008)

# Dimensions of Statistical Inference

- **Quantitative/qualitative:** quantifying the likelihood of an outcome (deriving explicit probability that observed result happened by chance) or making judgements that an observed result is surprising or unlikely

- **Closed-form formulas/simulation:** closed-form formulas (grounded in simplifying assumptions about underlying theoretical distributions) or estimating probabilities of a result through repeated simulations

**(Makar & Rubin, 2018)**

# Dimensions of Statistical Inference

- **Diversity of images of distributions:** single image of theoretical standard normal distribution or other approaches that tend to have more pictures, illustrating empirical data distributions (often non-normal, with superimposed visualizations)

- **Choices of measures of central tendency and variability:** using mean and standard deviation as measures or using measures such as the median and IQR

- **Community acceptance:** traditional formal statistical inference or informal approaches to statistical inference

**(Makar & Rubin, 2018)**

# Resources for Session

You can find all of the handouts and supporting CODAP links for this session at:

go.ncsu.edu/uscots2019

# Collecting Cane Toads Task

# Collecting Cane Toads Sample

- Come to the front of the class and take a sample of 50 cane toads. Count the number of female toads (red beads).
- Record your proportion at this link:
  - go.ncsu.edu/canetoadsample
- Let's look at everyone's samples
- Based on the samples of our class, what do you think the true proportion of female cane toads is in the wild?

# CODAP Simulation

Go to the following link

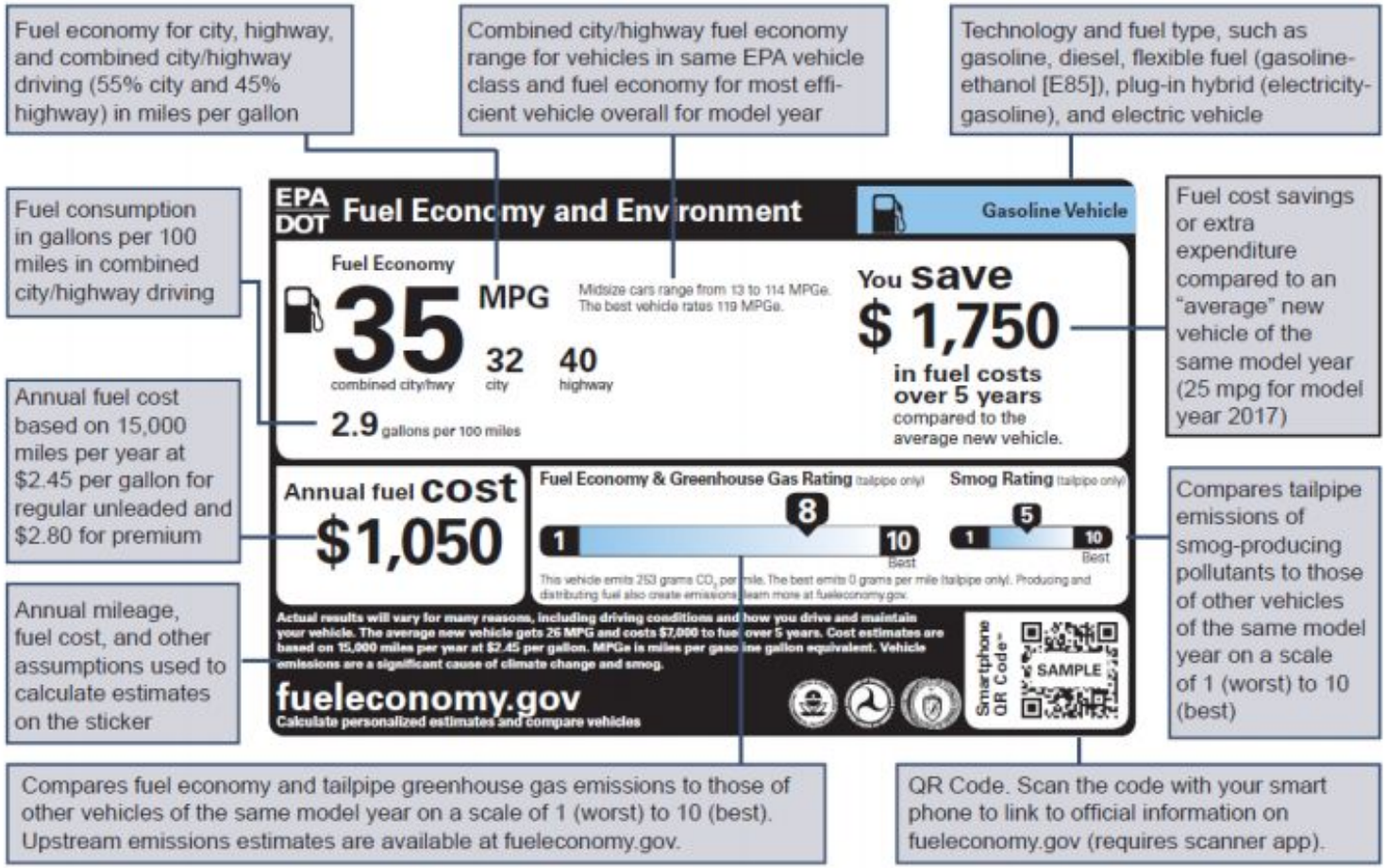
Sampler is found at go.ncsu.edu/uscots2019

# Investigating $CO_2$ Emissions of Vehicles

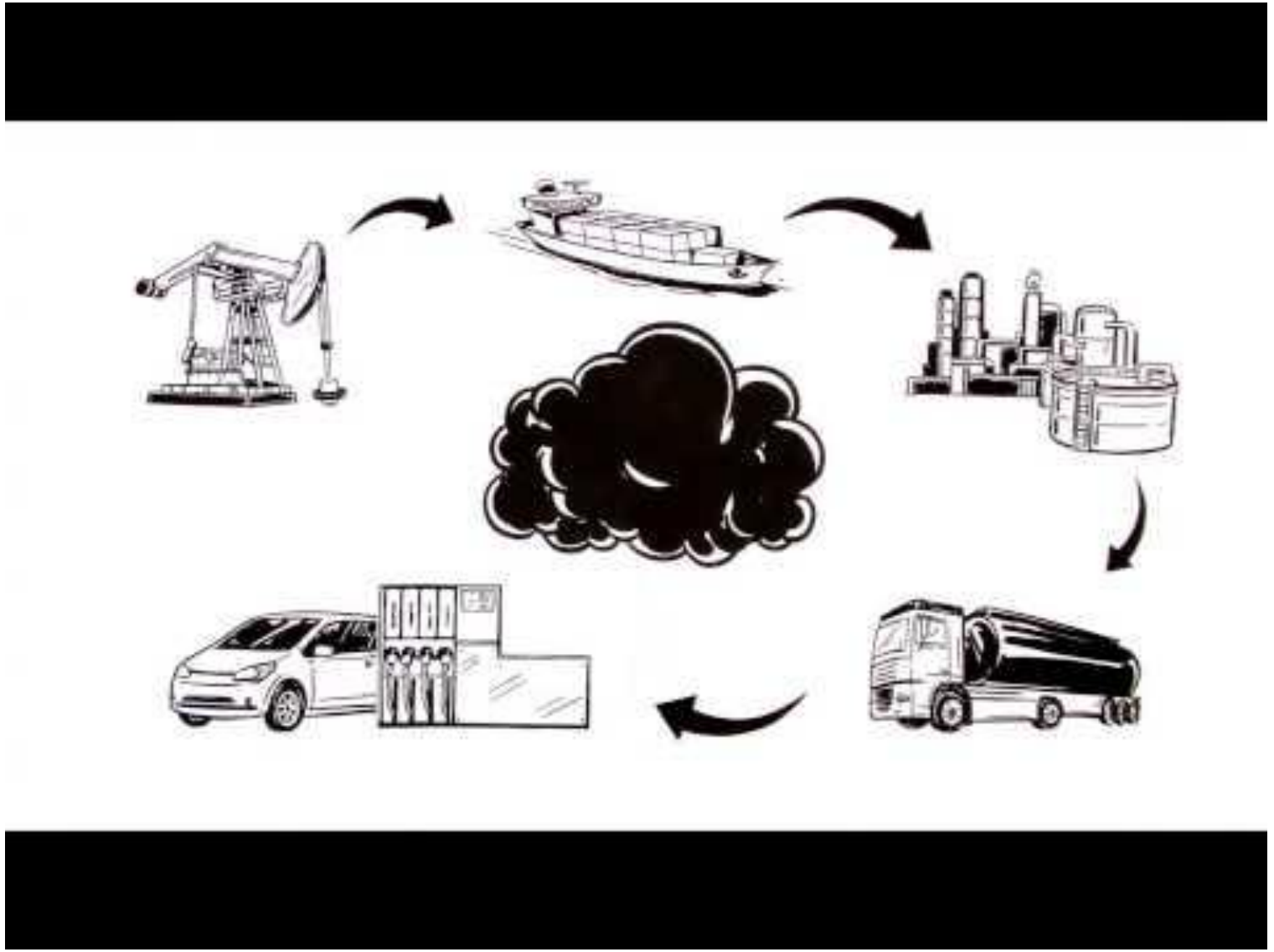
# Investigating $CO_2$ Emissions of Vehicles

## GETTING TO KNOW THE NEW FUEL ECONOMY AND ENVIRONMENT LABEL

The diagram below shows a sample label for a gasoline vehicle. Slightly different designs are used for flexible-fuel vehicles, electric vehicles, and plug-in hybrids. For more in-depth descriptions of label information for all vehicle types, visit **www.fueleconomy.gov**.



# Introduction to $CO_2$ Emissions



# Investigating $CO_2$ Emissions

The EPA claims that the average $CO_2$ emissions for passenger vehicles is about 404 grams per mile.

How could we test this claim?

# Going Beyond a few cases

Draw a sample of 50 vehicles using the CODAP sampler found at

go.ncsu.edu/uscots2019

From your sample can you make an inference….

# Schoolopoly
## Investigating the Fairness of Dice

# Context of Schoolopoly

A local school in your district is planning to create a board game similar to Monopoly that uses 6-sided color dice, rather than a regular die with 1-6. The game, called Schoolopoly, will be sold as part of a fundraiser. Each side of a die is a different color: black, blue, green, yellow, white, or red. Several companies are competing to produce the color dice that will be used in the board game. There is a rumor that one or more of the companies have been selling poor quality dice. If these rumors are indeed true, these companies should be avoided to ensure that the dice used for the game are “fair”. The school board is trying to decide which company should receive the contract for supplying the dice.

Three companies have provided a simulation of a sample of die from their factory.

- Dice R' Us
- Pips and Dots
- High Rollers, Inc.

# Investigation

Investigate whether or not the dice model obtained from each company is fair. You will use three separate CODAP documents to conduct simulations of rolling a die from each company using the Sampler. Your goal is to make a recommendation (i.e., a claim) as to whether dice should be purchased from each of the companies.

Your recommendation should be in the form of a letter to the school board. In the letter, you need to support your recommendation by explicitly including the following information about each of the three die companies:

1. Evidence as to whether each die company produces "fair" or "unfair" dice. This can include screenshots from your work in CODAP.
2. A description of your model for the probability distribution of the six possible outcomes of a die roll from each of the three companies (i.e., estimated probability of each possible outcome occurring for each die company).

**Part 1:** Collect data using the Sampler, a simulation tool, in CODAP. You do not need to write a letter, but respond to #1 & 2.

Dice R' Us

Pips and Dots

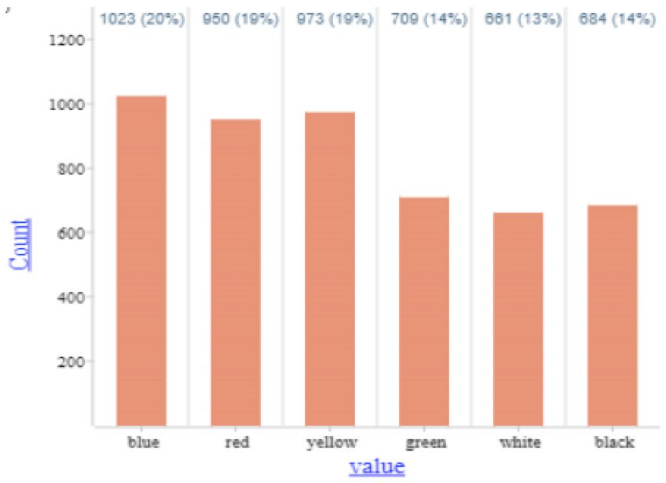
High Rollers, Inc.

**Part II:** After you have described a model for the probability distribution of the six possible outcomes of a die roll from each of the three companie (#2), use the Sampler in CODAP to build a model based on the probability distribution. Do you think you described the probability distribution?

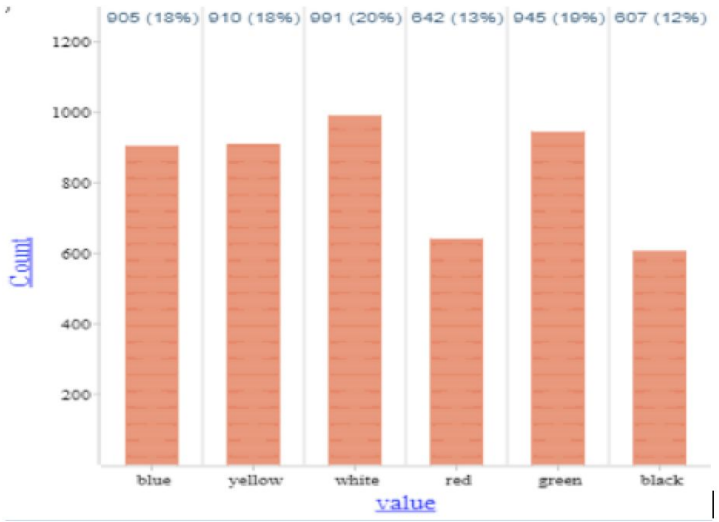
# One Student Response

"I have looked into this and have sampled dice from a few companies. The results amazed me. The company High Rollers Inc. sold dice that is nearly twice as likely to roll on red, blue, and yellow rather than green, white, and black. The company Dice R' Us is also not quite half as likely to roll on black and red. Yet, I did find that Pips and Dots is close in fairness. I ran a sample of 1000 rolls for each company."

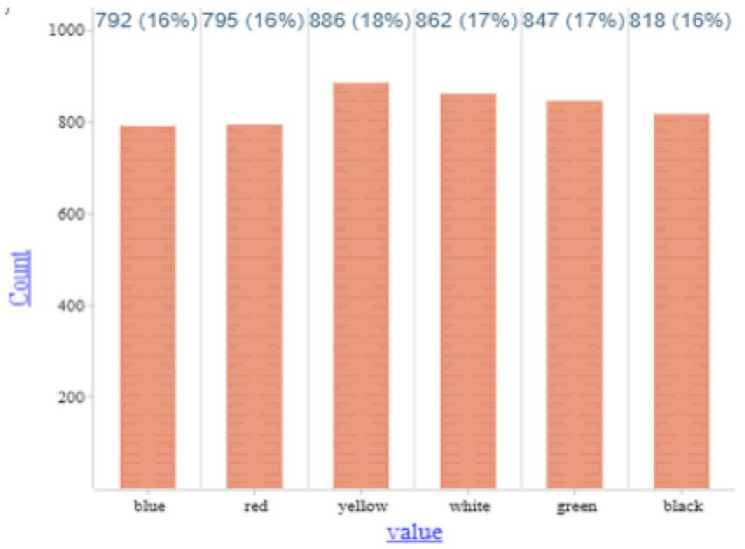
High Rollers Inc.




Dice R' Us




Pips and Dots



# Reflecting on the Tasks

What opportunities does each task provide in supporting inferential reasoning through simulation?

How might your students develop arguments to support their claims in these tasks?

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